Linear RF Power Amplifier Design for CDMA Signals: A Spectrum Analysis Approach

The nonlinear effects of an RF power amplifier on CDMA signals are analyzed and an estimation of the out-of-band emission levels for CDMA signals in terms of the power transistor’s traditional nonlinearity parameters and third- and fifth-order intermodulation points (IP3 and IP5) as well as the power level and bandwidth of the signal is derived. This result is useful to system and component engineers in the design and testing of RF power amplifiers for CDMA wireless systems.

In recent years, CDMA has been recognized as one of the most efficient and reliable schemes for cellular radio communications. The CDMA scheme was adopted as a new wireless communication industry standard, IS-95, by the Electronic Industries Association (EIA) and the Telecommunications Industry Association (TIA) in 1993 and possesses several distinct advantages. The spectral density is reduced and the RF resource is used more efficiently. Protection against co-channel interference is provided, which makes it possible to reduce the frequency reuse factor to 1. High communication security is also provided. Multipath effects are reduced significantly and channel manipulation and hand-over procedures are simplified greatly.

Many CDMA cellular networks are being deployed rapidly in the US and other countries. As in other communication systems, one of the critical and costly components in a CDMA system is the RF power amplifier. One of the main concerns in such an RF power amplifier design is the nonlinear effect of the amplifier. The nonlinearity of an RF amplifier can degrade the quality of the CDMA signal, increasing bit error rate and interference to adjacent channels. As part of the IS-95 standard, EIA/TIA has set requirements for the control of the nonlinearity of RF amplifiers used in cellular CDMA systems. The nonlinearity control is specified by the out-of-band power emission levels. Out-of-band emission is also called spectrum regrowth. A typical illustration of spectrum regrowth through an

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RF power amplifier is shown in Figure 1. The displayed output CDMA signal shows the effects of the nonlinearity of the RF amplifier assuming the gain of the amplifier is 1.

Traditionally, the nonlinearity of an RF amplifier is described by IP3 or, equivalently, by the 1 dB compression point.\(^3\)\(^,\)\(^4\) In experiments and analysis it was discovered that, in some situations, using IP3 only is not enough to describe the spectrum regrowth, especially when the fifth-order intermodulation is relatively high compared to the third-order intermodulation. Quantitatively, no clear relationship or expression exists to date between the out-of-band emission level and the traditional amplifier nonlinearity description. The lack of such a relationship poses difficulties for RF designers choosing components. This problem is generic in the design of RF power amplifiers for nonconstant envelope digital modulations.

In this article, based on the nature of the CDMA signal, explicit expressions are derived relating an estimation of the out-of-band power emission levels of an amplifier to its nonlinear parameters (IP3 and IP5) where IP5 is a parameter defined in a similar manner as IP3 to describe the fifth-order intermodulation quantitatively.

The bandwidth of pseudonoise code is much larger than that of the baseband signal. Hence, the bandwidth of the SS signal is determined by \(B\), the bandwidth of pseudonoise code. The signal \(s(t)\) of Equation 1 may be written as

\[
s(t) = s(t)\cos(2\pi f_0 t + \theta(t))
\]

where

\[
m_i(t) = \text{i}th \text{ baseband quadrature phase-shift keying or binary phase-shift keying modulated signal}
\]

\[
c_i(t) = \text{i}th \text{ pseudonoise binary code with a bandwidth of } B
\]

\[
f_0 = \text{carrier center frequency}
\]

\[
\theta_i(t) = \text{phase of the carrier associated with the } i\text{th SS signal}
\]

The bandwidth of pseudonoise code is much larger than that of the baseband signal. Hence, the bandwidth of the SS signal is determined by \(B\), the bandwidth of pseudonoise code. The signal \(s(t)\) is a sum of \(n\) SS signals where \(n\) is between nine and 64.

Each SS signal is one of the traffic or control channels. To study the spectral behavior of the CDMA signal, each SS signal is considered as a stochastic random process with zero mean. The \(s(t)\) of Equation 1 may be written as

\[
s(t) = s(t)\cos(2\pi f_0 t + \theta(t))
\]

where

\[
s(t) = \left[\sum_{i=1}^{n} m_i(t) c_i(t) \cos(\theta_i(t))\right]^2 + \left[\sum_{i=1}^{n} m_i(t) c_i(t) \sin(\theta_i(t))\right]^2 \frac{1}{2}
\]

and

\[
\theta(t) = \tan^{-1}\left(\frac{\sum_{i=1}^{n} m_i(t) c_i(t) \sin(\theta_i(t))}{\sum_{i=1}^{n} m_i(t) c_i(t) \cos(\theta_i(t))}\right)
\]

According to the law of large numbers and the central limit theorem in statistics,\(^9\) no matter what the distribution of each SS signal is, as \(n\) becomes large, the CDMA signal \(s(t)\) will be a bandlimited Gaussian stochastic process with zero mean. Using this argument, an equivalent band-limited white Gaussian process is utilized to study the spectral behavior of the output CDMA signal from an amplifier. The Gaussian stochastic signal has been studied extensively in signal processing. Many well-known results are used directly in the following derivation, making it possible to obtain the final result in a closed form.

A bandpass Gaussian statistic equivalent of \(s(t)\) may be expressed as

\[
s(t) = s(t)\cos(2\pi f_0 t + \theta)
\]

In this equation, \(s(t)\) is a Gaussian wide-sense stationary process with a power spectral density (PSD) of
and \( \theta \) is an arbitrary initial phase with a value that does not affect the statistic behavior of \( s(t) \).

**A Power Amplifier's Mathematical Model**

Clearly, an RF power amplifier is not a linear device, meaning that the output of the amplifier is not a scaled copy of the input signal. Considering an amplifier as a functional box, it can be modeled by a Taylor or Volterra series.\(^\text{10,11}\) The Taylor series model is valid for a memoryless nonlinear function. A memoryless amplifier implies that the output and input of the amplifier have one-to-one mapping. For an amplifier with only a few stages, a Taylor series model is fairly good for predicting the nonlinearity. Hence, the Taylor series is adopted for modeling RF power amplifiers. Using the equivalent input signal \( s(t) \) (determined in Equation 5), the output of an amplifier generally can be written as

\[
y(t) = O\{s(t)\} = F[s(t)]\cos\{2\pi f_0 t + \theta + \phi[s(t)]\} \tag{7}
\]

where

- \( O\{\cdot\} \) : operation of the power amplifier
- \( F[\cdot] \) : amplitude-to-amplitude conversion (AM/AM)
- \( \phi[\cdot] \) : amplitude-to-phase conversion (AM/PM)

The functions \( F[\cdot] \) and \( \phi[\cdot] \) are dependent on the nonlinearity of the amplifier and modeling type.

An important result from the Taylor series (or memoryless nonlinear) model is that there is no AM/PM conversion.\(^8\) Therefore, Equation 7 becomes

\[
y(t) = F[s(t)] \cos\{2\pi f_0 t + \theta\} \tag{8}
\]

If

\[
y(t) = F[s(t)] \tag{9}
\]

then the Taylor expansion of \( O\{s(t)\} \) can be used to determine \( y(t) \). Generally, the Taylor model of an RF amplifier can be written as

\[
y(t) = \sum_{i=0}^{\infty} a_i s^{2i} (t) \tag{10}
\]

where \( a_i \) is related to the gain \( G \) and linearity of the amplifier. Here, only the odd-order terms in the Taylor series are considered. Since the spectra generated by the even-order terms are at least \( f_0 \) away from the center of the pass band and for \( B << f_0 \), the effects from these terms on the pass band are negligible. Furthermore, as a linear amplifier, the third- and fifth-order terms dominate in Equation 10 for distortion. Therefore, in this analysis, the following model is used for an RF amplifier:

\[
y(t) = a_1 s(t) + a_3 s^3(t) + a_5 s^5(t) \tag{11}
\]

Substituting the input pass-band signal \( s(t) = \tilde{s}(t)\cos\{2\pi f_0 t + \theta\} \) into \( y(t) \) of Equation 10 (after manipulation) produces

\[
y(t) = y(t)\cos\{2\pi f_0 t + \theta\} \tag{12}
\]

where

\[
y(t) = a_1 \tilde{s}(t) + a_3 \tilde{s}^3(t) + a_5 \tilde{s}^5(t) \tag{13}
\]

with

\[
a_1 = a_1, \quad a_3 = \frac{3}{4} a_3 \quad \text{and} \quad a_5 = \frac{5}{8} a_5 \tag{14}
\]

Here, the coefficient \( a_1 \) is actually the linear gain \( G \) of the amplifier and the coefficients \( a_3 \) and \( a_5 \) are directly related to \( IP_3 \) and \( IP_5 \), respectively. For an amplifier with gain compression (\( a_3 < 0 \)), the expressions for these coefficients become\(^4,12\)

\[
a_3 = \frac{3}{4}10^{-\frac{G}{20}} \tag{15}
\]

\[
a_3 = \frac{5}{8}10^{-\frac{G}{20}} \tag{16}
\]

\[
a_5 = \frac{10^{-\frac{G}{20}} - \frac{G}{4}} {10^{-\frac{G}{20}} + \frac{G}{4}} \tag{17}
\]

From Equations 12, 15, 16 and 17, it can be seen that an amplifier’s output \( y(t) \) is a function of \( G, IP_3, IP_5 \) and the input signal \( s(t) \). Consequently, using Equation 12 and the stochastic property of \( \tilde{s}(t) \), the spectrum of \( y(t) \) can be calculated and the power emission levels can be determined. Therefore, all of the nonlinear effects of the amplifier on the CDMA signals can be evaluated.

Before proceeding further, it is worth examining the methods for measuring the nonlinear parameters \( IP_3 \) and \( IP_5 \) for a given transistor or amplifier. For most power transistors, the normally obtainable \( IP_3 \) parameters are listed in data books. The actual \( IP_3 \) of an amplifier is usually measured using a two-tone test\(^4\) as shown in Figure 3. According to this test, \( IP_3 \) is calculated using

\[
IP_3 = P_t + \frac{IM_3}{2} \tag{18}
\]
where

\[ P_t = \text{power of the original tone signals at the output.} \]

This expression is derived directly from the geometric relation shown previously. Note that to accurately measure IP3, the tone signal's power \( P_t \) should be chosen low enough such that the fifth-order intermodulations are ignorable at the output.

Unfortunately, IP5 usually is not provided in data books. However, it also may be measured using the two-tone test. Similar to Equation 18, IP5 can be determined by

The IM5 measurement is shown in Figure 4. In this test, the power level of \( P_t \) is higher than that of the IM3 measurement so that IM5 can be measured reliably.

**Autocorrelation of the CDMA Output Signal**

Now, the power spectrum of \( y(t) \) can be calculated. The power spectrum of \( \tilde{y}(t) \) is determined first and then \( y(t) \) is determined using

\[ P_y(f) = \int_{-\infty}^{\infty} R_y(\tau) e^{j2\pi f \tau} d\tau \]  

(21)

By definition, \( R_y(\tau) \) is expressed as

\[ R_y(\tau) = E\{y(t)y(t+\tau)\} \]  

(22)

where

\[ E\{\cdot\} = \text{mathematical expectation of } \{\cdot\} \]

Because of the relationship of \( \tilde{y}(t) \) and \( \tilde{s}(t) \) shown in Equation 13, \( R_y(\tau) \) may be expressed as the autocorrelation \( R_s(\tau) \) of \( \tilde{s}(t) \) as

\[ R_y(\tau) = R_s(\tau) + R_s^3(\tau) R_s^5(\tau) \]  

(23)

where

\[ K = N_0 B \]

\[ R_s(\tau) = \frac{N_0}{2\pi} \sin(2\pi f \tau) \]

In deriving Equation 23, some handy statistics results are employed. Using this result, the spectrum of the CDMA output signal from an amplifier can be calculated.
Fig. 5 Theoretical spectrum of an amplified CDMA signal.

An example is used to visualize the derived spectrum $P_y(f)$: The plot shown in Figure 5 is constructed by choosing the output power as 28 dBm, $f_3$ as 50 dBm and $f_1$ as 45 dBm, respectively. The bandwidth $B$ of the CDMA signal is selected to be 625 kHz (half of 1.25 MHz). It should be noted that the power spectrum unit in the $y$-axis is dBm/MHz. When this calculated power spectrum is to be compared with the power spectrum measured from a spectrum analyzer, the dBm/MHz unit then must be converted to the analyzer's resolution bandwidth at which the measurement is conducted.

Several observations are made when inspecting Equation 24 and the data plot: In the pass band $|f - f_0| \leq B$, the first term $P_0/2B$ corresponds to the linear output power density. The remaining terms in the pass band are caused by the nonlinearity. In traditional terminology, those terms are the cross-modulation terms, which are added to the linear term. For a linear amplifier, the cross-modulation power usually is much lower than the linear output power. Therefore, the cross modulation does not affect the pass-band spectrum significantly.

In the band $B < |f - f_0| \leq 3B$, the nonzero power spectrum density is generated by third-order as well as fifth-order intermodulation. This result shows that the out-of-band spectrum density is determined completely by the intermodulation. (These out-of-band frequency components usually are called spectrum regrowth.) This region contains the most harmful out-of-band emission.

In the band $3B < |f - f_0| \leq 5B$, the nonzero power density is generated by fifth-order intermodulation only. It has been observed in measurements that if the effect of the fifth-order intermodulation is considered negligible or the output power $P_0$ is low, the spectrum regrowth in this band may be so small that it is covered by noise.

In the band $|f - f_0| > 5B$, the power density is zero. This result is obtained because the signal was assumed to be bandlimited in the derivation. In practice, no bandpass signal is exactly bandlimited within $[-B, B]$, which means that in the band $|f - f_0| > 5B$, the power density is not exactly zero, but the emission in this band will very likely be covered by noise.

There is often a case where the fifth-order intermodulation does not dominate the out-of-band spectrum regrowth. Such is the case when an amplifier's output power is 5 to 10 dB lower than its 1 dB compression point or the fifth-order intermodulation tones in a two-tone test are more than 20 dB lower than the third-order intermodulation tones. In this situation, Equation 24 may be simplified significantly as

$$P_y(f) = \begin{cases} \frac{1}{2B} \left[ P_0 - 6P_0^2 10^{-\left(\frac{f - f_0}{B}\right)} + 9P_0^3 10^{-\left(\frac{f - f_0}{B}\right)} \right] & \text{if } |f - f_0| \leq B \\ \frac{3}{8} P_0^3 10^{-\left(\frac{f - f_0}{3B}\right)} \frac{1}{B^3} (3B - |f - f_0|)^2, & B < |f - f_0| \leq 3B \\ 0, & 3B < |f - f_0| \end{cases}$$

This simple result also makes it possible for a designer to estimate the spectrum regrowth when the $IP_5$ is not available.

With the explicit power spectrum of the output CDMA signal, the out-of-band spurious emission power may be calculated in a particular frequency band. It is this power that is used in IS-95 to specify the limit for the out-of-band emission control. To keep the result easy to use, only $IP_3$ is considered here.

Let a frequency band be defined by $f_1$ and $f_2$ outside the pass band. Using the results from $P_y(f)$, the emission power level within the band $(f_1, f_2)$, denoted as $P_{IM3}(f_1, f_2)$, can be determined easily by

$$P_{IM3}(f_1, f_2) = \int_{f_1}^{f_2} P_y(f) df = \frac{1}{8} P_0^3 10^{-\left(\frac{f_0}{3B}\right)} \frac{1}{B^3} \left[ (3B - |f_1 - f_0|)^3 - (3B - |f_2 - f_0|)^3 \right],$$

$$0 < f_1 < f_2, \quad B < |f_1 - f_0|, \quad |f_2 - f_0| \leq 3B$$

(26)

In most design procedures, a designer is concerned with the required $IP_3$ for a given out-of-band emission level. To obtain the desired $IP_3$, Equation 26 is solved for $P_{IM3}$ with the given $P_{IM3}(f_1, f_2)$, which yields

$$IP_3 = -5\log \left[ \frac{P_{IM3}(f_1, f_2) B^3}{P_0^3 \left(3B - |f_1 - f_0|\right)^3 - (3B - |f_2 - f_0|)^3} \right] - 4.52 \text{ (dBW)}$$

(27)

This result provides a direct relationship between the out-of-band emission power of a CDMA signal power amplifier and its $IP_3$. With a given required $IP_3$, the power amplifier design for a CDMA signal becomes a conventional RF power amplifier design.

DESIGN EXAMPLE AND COMPARISON WITH REAL MEASUREMENTS

In this example, the result shown in Equation 27 is used to design an 18 W amplifier, which complies with the out-of-
band emission level control requirement proposed for CDMA amplifiers. The two out-of-band emission level controls required in IS-95 are given as follows: The total CDMA signal bandwidth is 1.25 MHz; therefore, \( B = 625 \text{ kHz} \). In the band of \((f_0 + 750 \text{ kHz})\) to \((f_0 + 1.98 \text{ MHz})\), the suppression level between the output power and emission power at 30 kHz bandwidth must be larger than 45 dBc.

In the band mentioned previously \((f_0 + 1.98 \text{ MHz})\), the suppression at 30 kHz bandwidth is at least 60 dB. For this amplifier, \( P_0 = 18 \text{ W} \). For the \((f_0 + 750 \text{ kHz})\) to \((f_0 + 1.98 \text{ MHz})\) band, the corresponding maximum \( P_{IM3}(f_1, f_2) \) is expressed as

\[
P_{IM3}(f_1, f_2) = 18 \times 10^{-46} \times 18 = 4.52 \times 10^{-4} \text{ (W)}
\]

For the worst case, \( f_1, f_2 \) are assumed at the lower edge of \((f_0 + 750 \text{ kHz})\), \((f_0 + 1.98 \text{ MHz})\), that is

\[
f_1 = f_0 + 750 \text{ kHz} = f_0 + 0.75 \text{ MHz}
\]

and

\[
f_2 = f_0 + 750 \text{ kHz} + 30 \text{ kHz} = f_0 + 0.78 \text{ MHz}
\]

Then, from Equation 27, the required \( IP_3 \) becomes

\[
IP_3 = -5 \log \left[ \frac{0.000452 \times 0.625^3}{18^3 (1.875 - 0.75)^3 - (1.875 - 0.78)^3} \right] - 4.52
\]

\[
= 29.32 \text{ dBW} = 59.32 \text{ dBm}
\]

For the band described previously \((f_0 + 1.98 \text{ MHz})\), due to the fact that only third-order intermodulation is counted in the analysis, no significant out-of-band emission power at a frequency 3B away from \( f_0 \) exists. Therefore, it is easier to comply with this condition than with the first one. The results mentioned previously indicate that, in order to meet the IS-95 requirement, the CDMA amplifier must have an \( IP_3 \) of at least 60 dBm.

Next, the analytic result is compared with measurements made on an RF power amplifier. The real CDMA signal is generated by an HP E2507A multiformat communications signal simulator system. The carrier frequency is 900 MHz and the signal output power from the amplifier is 24.5 dBm. The \( IP_3 \) and \( IP_5 \) of the device at the same power level are 45 and 42.5 dBm, respectively. The resolution bandwidth is chosen as 3 kHz. Figure 6 shows the power spectrum predicted from this analysis compared to the spectrum measured on an HP 8595E spectrum analyzer. The frequency axis is offset by 900 MHz. The measured spectrum agrees with the analytically predicted spectrum in both the passband and shoulder areas.

The predicted result using \( IP_3 \) only vs. using both \( IP_3 \) and \( IP_5 \) is shown in Figure 7. It can be seen clearly that a better fit exists when both \( IP_3 \) and \( IP_5 \) are used vs. \( IP_3 \) only.

Several measurements of out-of-band emission levels of CDMA signals have been taken by QUALCOMM using a PA9440 amplifier designed by Celwave, Corvallis, CA. In these amplifiers, the fifth-order intermodulations are more than 25 dB lower than the third-order signals, thus the out-of-band emission level \( P_{IM3}(f_1, f_2) \) is calculated using Equation 26 with \( IP_3 \) only, which implies that the three-term Taylor series model is adequate for this case. The differences in decibels below the carrier power between \( P_0 \) and the average 30 kHz band emission level \( P_{IM3}(f_1, f_2) \) of four different amplifiers at \((f_0 + 750 \text{ kHz})\) are shown in Figure 8. The differences of \( P_0 \) and the emission levels calculated by Equation 26 are also shown. The \( IP_3 \) of the calculation is 44 dBm, which is an averaged value of the amplifiers. As shown, the actual measurements and analytical results are very close for \( P_0 > 20 \text{ dBm} \). The difference occurs for \( P_0 < 20 \text{ dBm} \), which is due to the fact that \( P_{IM3}(f_1, f_2) \) is not significant and is comparable to the noise floor. These results verify the analytical results discussed previously.

**CONCLUSION**

In previous work\(^5,6\) it was assumed that the effects of the fifth- or higher order intermodulations should be ignored. However, if the output power is high and the signal bandwidth is wide, the out-of-band emission power levels caused by fifth-order intermodulation could be significant.

In this article, by proposing a theoretical method to predict the output power spectrum of a CDMA power
amplifier, the traditional linearity parameter $IP_3$ and additional parameter $IP_5$ are linked directly with out-of-band emission levels. This analysis makes it possible for amplifier designers to use a conventional approach to design RF power amplifiers for CDMA signals. In addition to the results presented in this article, this derivation approach can be applied to out-of-band emission-level analysis for other communication standards.

ACKNOWLEDGMENT
The authors wish to thank Martina Testa and Robert Larkin of Celwave for their encouragement in the continuous study of nonlinear effects, including fifth-order intermodulation, as well as their support in making the measurements.

References
Linear RF power amplifier design for wireless signals: A spectrum analysis approach. Conference Paper. May 2003. We analyse the nonlinear effect of an RF power amplifier on CDMA signals and then derive an expression of an estimation of the out-of-band emission levels for CDMA signals in terms of the power transistor's traditional nonlinearity parameters, the third-order intermodulation coefficient (IP$_3$) and the fifth-order intermodulation coefficient (IP$_5$), as well as the power level and bandwidth of the signal. This result will be useful to system and component engineers in the design and test of RF power amplifiers for cellular. The design requirements for RF power amplifiers include high output power, high linearity, and good efficiency. CDMA systems are experiencing tremendous growth in the global communication marketplace. This growth is fueling many research programs and expanding the technology opportunities for all manufacturers. Amplifier designers have to make difficult choices when optimizing a design for a specific power amplifier. In certain cases, designers select output power as the key power-amplifier parameter to optimize. The receiver for this type of measurement is typically a spectrum analyzer or vector signal analyzer. Power meters are an integral part of any test setup due to their exceptional accuracy in measuring output power. RF power amplifier designs must be able to meet these challenges at reasonable cost if they are to be profitable. Therefore, given today's market demands and state-of-the-art amplifier design principles, the objective of this project is to analyze NXP's BLF6G20LS-140 LDMOS transistor and use it towards the design of an AB-class RF power amplifier. The design must meet several exacting specifications, but of particular interest will be to maximize video bandwidth. Analyses will rely on computer simulations using Agilent's Advanced Design System (ADS) and a transistor model provided by NXP Semiconductors. Once complete, the amplifier prototype will be evaluated in terms of its strengths and weaknesses in amplifying two-carrier CDMA signals of varying carrier spacing.